Estimation of seismic quality factor Q for Victoria, Australia

J. Wilkie1 & G. Gibson2

The effective quality factor Q for S wave attenuation in the Victorian lithosphere has been calculated using both the spectral ratio and coda Q methods. The frequency range studied was 2-20 Hz, and Q was found to be frequency dependent. A simple model has been adopted, with a low Q in the top 4 km of the crust that varies from 20 at a frequency of 1 Hz to 700 at 10 Hz. Empirical formulae for Q determined by linear regression of logarithmic plots of amplitude versus frequency were

\[ Q = 20f^{-0.5} \text{ for the 4 km thick upper crust, and } Q = 100f^{-0.85} \text{ for the lower crust, where } f \text{ is frequency}. \]

Australia. It is unusual for surface waves to be associated with small Victorian earthquakes, except for very shallow events. The coda of local earthquakes is, therefore, substantially scattered S-waves, and \( n = 1.0 \) was adopted in the Victoria region (Wilkie et al. 1993).

There is strong evidence from many sources (Aki & Chouet 1975; Singh & Herrman 1983; Woodgold 1990) that the value of Q varies with frequency. It combines the effect of aseismic properties of the medium, which may be frequency independent, and a frequency-dependent component due to scattering (Kvamme & Havskov 1989). The scattered energy depends on the distribution and size of crustal inhomogeneities and is, therefore, frequency dependent.

The common form of the relation between Q and frequency is a power law \( Q = Q_0 f^n \), where \( Q_0 \) is a reference Q at a frequency of 1 Hz and \( n \) is a numerical constant. Singh & Herrman (1983; Kvamme & Havskov 1989; Ambeh & Fairhead 1989). Both Kvamme & Havskov (1989) for Norway and Ambeh & Fairhead (1989) for West Africa obtained values of \( n \) close to 1, i.e. a linear relationship between Q and frequency. Ibanez et al. (1990) found \( n \) to range between 0.81 and 0.89 in southern Spain. However, values of \( n \) as low as 0.3 for Eastern Canada (Woodgold, 1990) have been found and Lindley & Archuleta (1992) determined negative values of \( n \) in California, i.e. Q decreasing with frequency.

Methods of analysis

Spectral Ratio Method

Expression (1) can be rewritten

\[ A(f,R) = \frac{E(f)}{R^n} e^{-tf} \]

(3)

where \( t \) is the travel time of the wave and \( n = 1 \). This is a very simple model for attenuation about a point source in a homogeneous, isotropic, infinite space, so the following analysis clearly has limitations.

The spectral ratio for a particular frequency, \( f \), is

\[ A_1 = \frac{R_2}{R_1} e^{-\frac{m(f)(t_2-t_1)}{Q}} \]

(4)

where \( R_1, R_2 \) and \( t_1, t_2 \) are the hypocentral distances and travel times of P or S phases recorded at seismograph sites 1 and 2. \( A_1 \) and \( A_2 \) are the respective spectral amplitudes. Taking natural logarithms of both sides of expression (4) we obtain

\[ \text{expression} \]

1 Department of Applied Physics, Victoria University of Technology, PO Box 14428 MMC, Melbourne, Victoria 3000, Australia

2 Seismology Research Centre, Royal Melbourne Institute of Technology, Plenty Rd, Bundoora, Victoria 3083, Australia
Spectral amplitudes were then measured at 4, 5, 10, 15 samples (2.56 seconds) of each S phase was Fourier transformed, passed through a 2 Hz high-pass filter and an anti-alias filter at 25 Hz. An interval of 256 sampling rate was 100 samples per second, in most cases the seismograph site s used are shown in Figure 2. The earthquakes used in this analysis were digitally recorded by the Seismology Research Centre network; the assigned magnitude was the mean of magnitudes available from the Victorian network and neighbouring seismograph networks. Each spectral amplitude was multiplied by the exponential of the site correction to correct for amplification effects of local geology.

The data available for use in the analysis were from relatively small earthquakes, and no fault-plane information was available. Hence, the radiation patterns are unknown. A partial solution to this problem would be to use colinear epicentres and seismograph sites, but such favourable geometry proved to occur infrequently. It is assumed that the radiation patterns have a random effect. About 500 values of Q were calculated, where possible, for the events listed in Table 1.

Using a power-law relation between Q and frequency, $Q \propto f^\eta$, and $\eta$ can be determined using linear regression with the expression

$$10^\log Q = 1.78 \pm 0.85 \log f \quad (8)$$

The errors are calculated assuming a log-normal distribution.

To limit the influence of the excessively large values of Q due to the lack of correction for radiation patterns and focussing/defocussing on the analysis, the harmonic mean was calculated at each frequency and its logarithm is plotted against the logarithm of frequency in Figure 4. The line of best fit in this case gives the relation

$$\log Q = 2.0 (\pm 0.2) + 0.8(\pm 0.2) \log f \quad (9)$$

For the Bradford Hills series of earthquakes, the Upper Yarra seismograph group (POL, VPE, MCV) and the Thomson Reservoir seismograph group (SIN, MAL, TOD, MIC, PAT, TMD, ABE) are approximately in line with the epicentres. Values of Q were calculated for each earthquake using average amplitudes at the two seismograph groups. Log Q versus log f and the corresponding regression line are plotted in Figure 5. Q values are of the same order of magnitude but lower than those given by expressions (8) and (9). The value of $\eta$ in this case is 1.0, i.e. a linear relation between Q and f: $Q = 27f$.

**Coda Q Method**

Aki (1969) and Aki & Chouet (1975) derived a formula for coda amplitude, assuming single or forward scattering. By determining the shape of the envelope of the coda for a particular frequency versus time, a value of Q can be obtained at that frequency. It is assumed that the coda consists of singly scattered waves from randomly distributed diverse scatterers in an isotropic medium and the earthquake source and seismograph are at the same location. Woodgold (1990) and Ambeh & Fairhead (1989) applied a correction for the separation of source and
receiver. However, the corrections in this study are small compared with errors in the estimation of the coda envelope and, consequently, are ignored.

The power spectral density for body waves at time $t$ after the origin time (lapse time) can be written (Aki & Chouet 1975; Woodgold 1990) as:

$$ P(\omega, t) = E(\omega) t^{-2} e^{-\frac{t}{Q}} $$  \hspace{1cm} (10)

where $E(\omega)$ is a source spectrum. The amplitude spectrum is then

$$ A = E_A(\omega) t^{-1} e^{-\frac{t}{20}} $$  \hspace{1cm} (11)

where $E_A(\omega)$ is again a source term.

By taking the amplitude at different lapse times, $t_1$ and $t_2$, the amplitude ratio can be written

$$ \frac{A_1}{A_2} = \frac{t_2}{t_1} e^{-\frac{t_2-t_1}{Q}} $$  \hspace{1cm} (12)

Thus, by taking the amplitude of the coda envelope for a specified frequency at differing lapse times, $Q$ can be determined at that frequency.

In practice, the digitised seismograph records were band-pass filtered, using zero-phase (acausal) filters in the bands: 3–7, 7.5–12.5, 12.5–17.5, 17.5–22.5 Hz, the bands being centred on 5, 10, 15 and 20 Hz. In each frequency band the coda envelope amplitude was estimated by eye and $Q$ calculated using

$$ \ln A_1 + \ln t_1 - \ln A_2 - \ln t_2 = \frac{\ln(t_2-t_1)}{Q} $$  \hspace{1cm} (13)

At short range (less than 17 km, mean 11.6 km), coda $Q$ is significantly different from $Q$ determined from the spectral ratio method. Using $Q = Q_0 f^\alpha$ in the form of expression (7), Figure 6 was obtained with the corresponding regression expression

$$ \log Q = 1.28 + 0.45 \log f $$  \hspace{1cm} (14)

Assuming a log-normal distribution, the standard errors in the slope and intercept were calculated giving

$$ \log Q = 1.28 \pm 0.45 \log f $$  \hspace{1cm} (15)

The standard deviation of the residual error in $\log Q$ was 0.25.

### Evidence from source spectral analysis

The idealised spectrum of an earthquake consists of constant values at low frequencies, and above the corner frequency the spectral amplitude decreases as a power of frequency. The theoretical value of this power is $-2$ (Brune 1970, 1971) and the spectra of large earthquakes usually agree with that value (Wyss & Hanks 1972). However, for small earthquakes larger negative values often occur (Fletcher 1980).

Observed spectra—after corrections have been made for
radiation pattern, geometrical spreading, attenuation, site amplification, and free surface—agree with the idealised shape and can be used to calculate the moment of the seismic source. If the correction for attenuation is accurate, then the spectral decay beyond the corner frequency will be independent of range R.

The spectral decay gradient of S wave spectra, corrected for attenuation, then plotted against range, was used as a check of \( \eta \), the exponent giving the frequency dependence of Q. Three 1991 earthquakes (March 28, May 01 & May 02; Table 1) were used. The combined gradient versus range data are plotted in Figure 7(a–d) for respective values of \( \eta \) of 0.7, 0.8, 0.9, and 1.0. For all seismograms, 2.56 seconds of recording from the time from the S wave arrival were Fourier transformed and log spectral amplitude versus log frequency was plotted on a computer graphics display. The computer operator then manipulated lines representing a simplified source spectrum to determine the spectral amplitude at low frequency, the corner frequency, and the spectral decay gradient at higher frequencies. From Figure 7 the trend

<table>
<thead>
<tr>
<th>Date</th>
<th>UT</th>
<th>Place</th>
<th>ML</th>
<th>Longitude</th>
<th>Latitude</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988-3-11</td>
<td>2253</td>
<td>Maldon</td>
<td>3.2</td>
<td>144.045</td>
<td>-37.043</td>
<td>10.8</td>
</tr>
<tr>
<td>1988-4-21</td>
<td>1645</td>
<td>Bunnaloo</td>
<td>3.2</td>
<td>144.463</td>
<td>-35.670</td>
<td>7.5</td>
</tr>
<tr>
<td>1988-5-20</td>
<td>1326</td>
<td>Rushworth</td>
<td>2.1</td>
<td>145.078</td>
<td>-36.608</td>
<td>6.0</td>
</tr>
<tr>
<td>1988-7-3</td>
<td>0823</td>
<td>Bunnaloo</td>
<td>4.0</td>
<td>144.496</td>
<td>-35.672</td>
<td>3.6</td>
</tr>
<tr>
<td>1988-7-15</td>
<td>0306</td>
<td>Bunnaloo</td>
<td>3.2</td>
<td>144.501</td>
<td>-35.730</td>
<td>8.6</td>
</tr>
<tr>
<td>1989-8-25</td>
<td>1500</td>
<td>Echuca</td>
<td>3.2</td>
<td>144.588</td>
<td>-36.184</td>
<td>21.4</td>
</tr>
<tr>
<td>1989-10-7</td>
<td>0045</td>
<td>Euroa</td>
<td>2.8</td>
<td>145.573</td>
<td>-36.815</td>
<td>8.5</td>
</tr>
<tr>
<td>1991-3-19</td>
<td>0109</td>
<td>Eildon</td>
<td>2.4</td>
<td>145.789</td>
<td>-37.169</td>
<td>13.0</td>
</tr>
<tr>
<td>1991-3-28</td>
<td>2126</td>
<td>Boolara South</td>
<td>3.6</td>
<td>146.249</td>
<td>-38.525</td>
<td>20.6</td>
</tr>
<tr>
<td>1991-5-1</td>
<td>2239</td>
<td>Bradford Hills</td>
<td>2.8</td>
<td>144.086</td>
<td>-36.901</td>
<td>0.0</td>
</tr>
<tr>
<td>1991-5-2</td>
<td>0113</td>
<td>Bradford Hills</td>
<td>2.7</td>
<td>144.087</td>
<td>-36.893</td>
<td>0.0</td>
</tr>
<tr>
<td>1991-5-3</td>
<td>1728</td>
<td>Bradford Hills</td>
<td>3.5</td>
<td>144.086</td>
<td>-36.898</td>
<td>1.5</td>
</tr>
<tr>
<td>1991-6-8</td>
<td>1925</td>
<td>Bradford Hills</td>
<td>3.0</td>
<td>144.110</td>
<td>-36.888</td>
<td>0.8</td>
</tr>
<tr>
<td>1991-11-2</td>
<td>2056</td>
<td>Glenthompson</td>
<td>2.6</td>
<td>142.493</td>
<td>-37.608</td>
<td>1.2</td>
</tr>
<tr>
<td>1992-1-26</td>
<td>1454</td>
<td>Bradford Hills</td>
<td>2.6</td>
<td>144.093</td>
<td>-36.878</td>
<td>5.3</td>
</tr>
<tr>
<td>1992-2-9</td>
<td>1043</td>
<td>Mt Buller</td>
<td>2.5</td>
<td>146.527</td>
<td>-37.158</td>
<td>9.8</td>
</tr>
</tbody>
</table>
of the least squares regression lines indicates that for $\eta = 0.9$ there is minimum variation of spectral decay gradient with range. This value of $\eta$ is close to that previously estimated from linear regression of harmonic-mean estimates of $Q$ in Figure 4 and expression (9):

$$Q = Q_0 \eta^l = 60^{0.85}$$  \hspace{1cm} (16)

The gradients in Figure 7 are not significantly different

Figure 7. The gradient of log spectral amplitude against log frequency (above the corner frequency) plotted against range, with attenuation corrections using values of $\eta$ of (a) 0.7, (b) 0.8, (c) 0.9, (d) 1.0.

from zero for $\eta = 0.85 \pm 0.1$.

The spectral amplitude at low frequency can be used to estimate $M_o$, the moment of an earthquake, and stress drop (Brune 1970, 1971).

In Figure 8, moment estimates are shown from spectra at individual recording sites for four events (indicated in Figure 8 and listed in Table 1) with similar magnitude.

Figure 8. Superimposed plots of the moments of four earthquakes, calculated for each recording site using $Q = 100^0.9$, against range. This shows a characteristic increase in range, which was also exhibited by all other analysed events.

The spectra have been corrected for attenuation using $Q = 100^{0.9}$. The moment estimate clearly increases with range between 100 and 150 km. A similar pattern is exhibited by other analysed events. This pattern could be explained by an increase of $Q$ with range as waves travel at greater depth. Another explanation is in terms of phase arrivals. Remember that 2.56 seconds following the onset of the $S$ arrival are used to estimate the spectrum. Up to a range of about 100 km, the $S$ coda will be dominated by the $S_g$ phase. According to Wesson’s (1988) velocity model, reflections off the Moho ($S_mS$) begin to arrive within 2.56 seconds of the $S_g$ phase at a range of about 120 km. The extra energy contained in the $S_mS$ phase would contribute to the apparent increase of moment with range evident in Figure 8. It is likely that both phase arrival and variation of $Q$ with depth contribute to the
moment pattern.

In Figure 9, the moments are again computed and corrected for attenuation, but this time allowing attenuation to increase with range (depth). Keeping η at 0.9, but varying Qo as follows:

\[ Qo = 100 \] to a range of 120 km,
\[ Qo = 100 + 1.25 \times (R - 120) \] for range R between 120 and 200 km, and
\[ Qo = 200 \] beyond 200 km range.

There is no significant variation of the moment estimates with range. The scatter in Figure 9 can be attributed to the lack of accounting for the source radiation pattern and focussing/defocussing effects.

If the attenuation is correctly accounted for, then calculated values of Mo can be used to estimate magnitudes. Relations between magnitude and Mo that can be used include that given by the US Geological Survey, National Earthquake Information Centre:

\[ Mw = (\frac{1}{2}) \log Mo - 10.7 \] (17)

where Mo is expressed in dyne-cm (Hanks & Kanamori 1979), or

\[ mb = \log Mo - 11.3 \] (18)

and \[ 1.4mb = \log Mo - 9.2 \] (19)

for western US earthquakes and eastern North American earthquakes respectively, where Mo is expressed in Nm, (Patton & Walter 1993).

Using Qo = 100 for the lower crust, the values of Mw, calculated using mean Mo values and expression (17), closely agree with the assigned Victorian local magnitude values (Table 2). The Mw scale was originally defined so that for smaller earthquakes the numerical values will be similar to those for the Mt. scale. Qo = 100 in the lower crust is consistent with estimates by the spectral ratio method. A mean stress drop of 1.8 MPa for the earthquakes used in the spectral amplitude analysis (Table 2) is consistent with values between 1 and 10 MPa expected for intraplate earthquakes (Nuttli 1983; Boatwright & Choy 1992).

<table>
<thead>
<tr>
<th>Event</th>
<th>Mo (MNm)</th>
<th>Mt</th>
<th>Mw (exp.18)</th>
<th>Stress drop (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 01 1991</td>
<td>1.85 \times 10^7</td>
<td>2.8</td>
<td>2.81</td>
<td>3.36</td>
</tr>
<tr>
<td>Jan 26 1992</td>
<td>1.20 \times 10^7</td>
<td>2.6</td>
<td>2.68</td>
<td>1.42</td>
</tr>
<tr>
<td>Feb 09 1992</td>
<td>0.68 \times 10^7</td>
<td>2.5</td>
<td>2.52</td>
<td>2.61</td>
</tr>
<tr>
<td>Nov 02 1991</td>
<td>1.17 \times 10^7</td>
<td>2.6</td>
<td>2.67</td>
<td>0.53</td>
</tr>
<tr>
<td>Mar 19 1991</td>
<td>0.54 \times 10^7</td>
<td>2.4</td>
<td>2.46</td>
<td>1.47</td>
</tr>
</tbody>
</table>

**Discussion**

The structure of the Victorian lithosphere is not known in detail; however, the surface structures have significantly different properties from the deeper lithosphere. Although there is considerable lateral variation with geological structures, the average depth of this low-velocity crustal region is 3.6 km in the model of the Victorian lithosphere, Wesson (1988). A simple two-layer model (Singh et al. 1982; Lindley & Archuleta 1992) can be adopted with a 3.6 km thick upper crust and a lower crust to an average depth of 36 km at the Mohorovicic discontinuity. Such a simple model is of sufficient accuracy for the methods used here to determine Q.

Q estimated from coda at close range is representative of the surface layer. The coda-Q method is, in theory, only applicable if earthquake and source are at the same location. However, by using events with an average range of 11.6 km and mean depth of 4.6 km, and using coda within a few seconds of the S phase arrival, the scattering volume was minimised and the values of Q obtained are representative of the higher attenuation, low Q, upper crustal region (Wennerberg 1993). The coda-Q analysis found that the Qs in the upper crust were given by Q = 200^{0.5}.

At greater distances (50–100 km) coda Q would be representative of a larger scattering volume and greater depth, and was found to be comparable with those obtained using the spectral ratio method.

The spectral ratio method can only determine Q in the region below the low-velocity upper crust (Singh et al. 1982). Two methods of data analysis were used: a linear least squares fit of individual Q estimates as a function of frequency in the log-log domain gave the relationship Q = 100^{0.75}; and a linear least squares fit of log Q (harmonic means of Q at specific frequencies ) and log f gave Q = 60^{0.85}. Both methods, one taking the mean of logarithms and the other using harmonic means, reduce the contribution of extreme Q values in the analysis. Both are applicable to the lower crust. Considering also the results from the source spectrum analysis and earthquake moments, it is felt that Q = 100^{0.85} is representative of attenuation in the lower crust from depths of about 4 to 36 km.

The variation of seismic wave amplitudes with distance is very complex, and in this study we have not been able to resolve between increasing Q with depth and the effect of larger phase arrivals at particular distances, such as SmS reflections beyond 120 km. The original purpose of this study was to determine Q for moment computations, and it has been found that consistent moments can be determined if Q is increased for distances between 120 and 200 km.

The attenuation models used here are very crude. Future studies of Q could include more realistic attenuation functions that allow variation of Q between and within layers. Corrections could be included for source radiation pattern, travel path including amplitude variation with distance of different phases such as SmS, and spectral site effects. The analysis should include simultaneous inversion by least squares rather than step by step inversion.

**Conclusions**

Both the coda Q and spectral ratio methods show that Q increases with increasing frequency. A crude crustal attenuation model has been derived for the frequency range 2 Hz to 20 Hz with Q = 200^{0.5} in the 4 km thick upper crust, and Q = 100^{0.85} in the lower crust to a depth of about 36 km. This gives a low Q in the top
Bakun, W.H. & Joyner, W.B., 1984. The ML scale in based magnitude for all local earthquakes. J. Wilkie wishes to thank both the Victoria Seismology Research Centre, RMIT, for access to data and the Director, G.Gibson, and staff of the Victorian Earthquake Information Centre, is encouraging. Accuracy considerations are complicated by the wide distribution of $Q$ values determined for each frequency and the distortion resulting from data uncorrected for radiation patterns and focussing/defocussing effects. However in both upper and lower crust a clear dependence of $Q$ upon frequency is evident. A progressive change in $Q$ with depth in the lower crust probably exists, but has not been resolved by this initial study.

The close correspondence between local magnitude values, $M_l$, and the values of $M_w$ calculated from moments that were computed using these $Q$ functions, and using the relationship adopted by the US Geological Survey, National Earthquake Information Centre, is encouraging. It implies that the local $M_l$ magnitudes computed for Victorian earthquakes are consistent with those in California. In future it will be possible to use a spectrally based magnitude for all local earthquakes.

**Acknowledgments**

The authors thank the referees for their detailed comments on the paper and considerable help with suggestions for presentation. J. Wilkie wishes to thank both the Victoria University of Technology for supporting the research with a grant of leave and the Director, G.Gibson, and staff of the Seismology Research Centre, RMIT, for access to resources and their generous cooperation.

**References**


