ITRF to GDA94 coordinate transformations

John Dawson and Alex Woods

Abstract. The Geocentric Datum of Australia 1994 (GDA94) is a static coordinate datum realised with respect to the International Terrestrial Reference Frame (ITRF) at the reference epoch of 1 January 1994. At this time GDA94 and ITRF were coincident, however, as a consequence of the tectonic motion of the Australian Plate, ongoing refinement of the ITRF and crustal deformation, the two reference frames have diverged and the absolute difference between them is now approximately 1 m. Consequently, precise coordinate transformations between ITRF and GDA94 are required for many applications, and in this study these transformations are reviewed, improved and extended. New transformation parameters between ITRF and GDA94 are computed, including the specific ITRF realisations of ITRF1996, ITRF1997, ITRF2000, ITRF2005 and ITRF2008. The two most recent ITRF realisations, ITRF2005 and ITRF2008, after transformation have a root-mean-square difference of less than 10 and 30 mm in the horizontal and vertical components, respectively, with respect to GDA94 at the Australian Fiducial Network (AFN) stations. However, the magnitude of some residuals exceed 15 and 60 mm in the horizontal and vertical components, respectively, which reflects the accuracy limit of GDA94. Finally, the implications and future strategies for managing the differences between GDA94 and ITRF are discussed, including novel coordinate transformation approaches and justifications for the modernisation of GDA.

Keywords. GDA94, ITRF, coordinate systems, coordinate transformations.

1. Introduction

The Geocentric Datum of Australia 1994 (GDA94) is the official Australian coordinate system and was implemented in 1995 by the Intergovernmental Committee on Surveying and Mapping (ICSM) (ICSM 1998). GDA94 is static, and its coordinates remain unchanged with time. In contrast, the widely used international coordinate system, the International Terrestrial Reference Frame (ITRF) (e.g., Boucher et al. 1993), is dynamic, and its coordinates change as a combination of both coordinate offsets and as linear functions of time primarily to account for unmodelled measurement biases and tectonic processes. Static coordinate systems are from most user perspectives easier to implement and maintain; whereas, dynamic coordinate systems allow station coordinates to be more accurately modelled over time.

GDA94 is itself based on the ITRF, specifically ITRF1992 (Boucher et al. 1993), and on 1 January 1994 the two were coincident, however, by 2010 they have diverged by over 1 m. Geoscience Australia has previously computed and distributed to users the transformation parameters between GDA94 and ITRF1996, ITRF1997 and ITRF2000 (Dawson and Steed 2004). These transformations have found wide application in the Australian spatial community, for example, the AUSPOS on-line GPS processing service (Dawson et al. 2002) uses them to deliver GDA94 coordinates from an ITRF based computation system.

The primary aim of this study was to improve and extend the GDA94 to ITRF coordinate transformations of Dawson and Steed (2004). Transformation parameters between GDA94 and the ITRF1996, ITRF1997, ITRF2000, ITRF2005 and ITRF2008 reference frames are provided. For the ITRF2005 and ITRF2008 cases these are the first published results. Additionally, a review of the impact of incorrectly dealing with inconsistencies between satellite trajectories and reference station coordinates when reducing differential GNSS observations in GDA94 based geodetic networks was undertaken. Then as an alternative to station coordinate transformations, ITRF-to-GDA94 satellite-trajectory transformations and their limitations are investigated. Finally, by highlighting the limitations of GDA94, the case for the modernisation of the geocentric datum of Australia is outlined.

2. ITRF and GDA94

Beginning in the early 1990s, and as global geodetic observing networks have densified and analysis techniques improved, the ITRF has been iteratively updated with better estimates of station coordinates and velocities. ITRF updates include, for example,

GDA94 was realised using Global Positioning System (GPS) data from both the Australian Fiducial Network (AFN) (Twilley and Digney 2001) and from a globally distributed set of fundamental geodetic stations from the International Global Navigation Satellite System (GNSS) Service (IGS), which was known at the time as the International GPS Service (Dow et al. 2008). GDA94 was connected to the ITRF by adopting the ITRF1992 coordinates of 13 fundamental stations in the IGS at the reference epoch of 1994 (Morgan et al. 1996). The metre level difference between GDA94 and ITRF at the present time is a consequence of: the tectonic motion of the rigid Australian Plate, which is approximately 70 mm yr$^{-1}$ in the NNE direction (Tregoning 2003); the differences between ITRF1992 and the later ITRF realisations, which are caused by modelling and input data differences and can be as large as 16 and 75 mm in the horizontal and vertical components of station position, respectively, and 5 mm yr$^{-1}$ in station velocity; and residual intra-plate, regional and local deformation, which is generally less than 1 mm yr$^{-1}$ in the horizontal components (Kogan and Steblov 2008) but for some sites can be as large as 6 mm yr$^{-1}$ e.g., the vertical component of the Perth IGS station (Altamimi et al. 2007).

The precise relationship between GDA94 and ITRF must be well understood because many Australian users by necessity work with ITRF based coordinate systems. There are two main reasons for this. The first is that the two most widely adopted satellite trajectory products used for positioning are ITRF based. Specifically, the satellite ephemerides broadcast by the GPS in the World Geodetic System 1984 (WGS-84) reference frame (NIMA 2000b,a) are aligned to ITRF2000 at the centimetre level (Merrigan et al. 2002) and the GPS and GLONASS orbits derived by the IGS are also ITRF based (Griffiths and Ray 2009). The second driver is that GNSS positioning services in the Australian region often integrate data from both stations located on the Australian continent and elsewhere in the Asia-Pacific region, e.g., the OmniSTAR service (Fugro 2009). They use ITRF to prevent degradation of the positioning accuracy that would occur as a result of the relative tectonic motion of the ground stations. As a consequence of these two drivers the ITRF has become a standard for GNSS analysis, and user access to GDA94 often remains as an additional issue to resolve with coordinate transformations.

3. Input solution datasets

In this section, the details of the input data sets used to determine the ITRF-to-GDA94 transformations are reviewed. The ITRF1996, ITRF1997, ITRF2000, ITRF2005 and ITRF2008 solutions were obtained from the ITRF product center of the International Earth Rotation and Reference System Service (IERS) in the Software INdependent EXchange (SINEX) format (Blewitt et al. 1994). They included the parameter estimates of the station coordinates and velocities and their corresponding full variance-covariance (VCV) matrix. For consistency, all the ITRF solutions were first projected using the station velocity estimates to the reference epoch of 1 January 1994 thus ensuring that the derived transformation parameters were referenced to the same epoch as GDA94. For the stations at Tidbinbilla, Yarragadee and Perth, which in some ITRF realisations have multiple coordinate estimates, i.e. they have one or more discrete offsets in their position time series, the coordinate estimate that was most valid for the reference epoch of 1994 was adopted.

The reference GDA94 coordinate values were obtained from the gazetted positions of the AFN (Commonwealth of Australia Gazette 1998), Figure 1, which include the 7 AFN stations: Alice Springs, Hobart, Tidbinbilla, Yarragadee, Karratha, Darwin and Ceduna. The Cocos Island and Macquarie Island stations from the computations were excluded because they are not located on the Australian Continent and also since 1994 have experienced large earthquakes, including the 2000 M7.9 Cocos Island Earthquake and the 2004 M8.1 Macquarie Island Earthquake (see the National Earthquake Information Center, NEIC, database). Observed coordinate offsets at these stations (e.g., Altamimi et al. 2007), at the times of these earthquakes, suggests significant co-seismic displacement and further justifies their exclusion from the reference coordinate set. The non-AFN IGS station in Perth (i.e. PERT), Western Australia, was used as it was included in the GDA94 computation and is consistent with the other gazetted AFN positions (Morgan et al. 1996). The Townsville AFN station TOW2 (i.e. Cape Ferguson), which in 1995 replaced the gazetted station at Mount Stuart (TOWA), was also used in the computation. These two stations were at this time con-
nected by a local GPS survey. In some cases all the GDA94 reference stations could not be used in the computations. Specifically, ITRF1996 and ITRF1997 did not include the Ceduna station, ITRF1997 did not include the Townsville station and ITRF1996 did not include the Darwin station.

4. Coordinate transformation method

A 14 parameter relation was adopted to transform station coordinates and velocities from ITRF to GDA94. This relation included three translations, \( t_x, t_y, t_z \), three rotations, \( r_x, r_y, r_z \), one scale factor, \( s_c \), and their first-time derivatives, namely \( \dot{t}_x, \dot{t}_y, \dot{t}_z, \dot{r}_x, \dot{r}_y, \dot{r}_z \) and \( \ddot{s}_c \). This transformation model can be expressed in equation (1), where \( T \{ \} \) is the ITRF-to-GDA94 transformation, \( [X_{\text{GDA94}}, Y_{\text{GDA94}}, Z_{\text{GDA94}}]^T \) is the vector of GDA94 geocentric coordinates (m), \( [X_{\text{ITRF}}, Y_{\text{ITRF}}, Z_{\text{ITRF}}]^T \) is the vector of ITRF coordinates (m) at some epoch (yr), \( t \) and \( t_0 \) is the reference epoch (yr) of the transformation parameters, which in this work was always 1994. The translations and their rates are expressed in m and m yr\(^{-1}\), respectively. The rotation and their rates are expressed in rad and rad yr\(^{-1}\), respectively. The scale is unit-less and the scale rate is expressed in yr\(^{-1}\).

This full fourteen parameter relation, equation (1), is developed from the standard seven parameter relation by differentiating it with respect to time and eliminating the negligible terms (Altamimi et al. 2002).

In practice, there are two different ways of applying the sign conventions for the rotations. For both cases the sign convention is the same, that is a positive rotation is an anti-clockwise rotation, when viewed along the positive axis towards the origin. The IERS assumes the rotations to be of the position around the coordinate axes while the method historically used in Australia assumes the rotations to be of the coordinate axes. The only difference in the transformation formula is a change of the sign of the rotations and their rates. In this work the method historically used in Australia was adopted.

In the absence of a national crustal velocity model, such as those available in a few other countries (e.g., New Zealand, Blick et al. 2005), there was no basis for adopting other, more sophisticated, transformation methodologies. As a consequence, the adopted transformation methodology cannot accurately represent regional or local deformation, or discrete deformation, such as that associated with earthquakes, which fortunately are rare in Australia (Dawson and Tregoning 2007). Additionally, it cannot adequately model the significant differences between the GDA94 solution (Morgan et al. 1996) and the latest ITRF realisations due to observation modelling improvements particularly in the height component of position, which are largely associated with the significant more recent improvements in antenna phase centre modelling (e.g., Schmid et al. 2007). However, this simple transformation model facilitates implementation by a broad user community, which is a significant benefit.

Dawson and Steed (2004) computed ITRF-to-GDA94 transformation parameters using only the diagonal terms of the input VCV matrices. To investigate the impact of this, three input solution weighting strategies were experimented with namely, unit weights, diagonal terms of the input VCV matrices and full VCV matrices. The differences between the weighting strategies were in general significant (at the 95% confidence level). As a test, the vertical component of station position and velocity of the input VCV matrices was down-weighted by a factor of 100; However, this did not significantly impact the results. Consequently, the full and unmodified VCV matrices were adopted for the computations. To estimate the transformation parameters, the pro-
5. Results

At the reference epoch of 1994, after transformation the root-mean-square (RMS) residual differences between GDA94 and the ITRF2005 and ITRF2008 solutions were less than 10 and 30 mm in the horizontal and vertical components, respectively. Table 1. For ITRF2000 the RMS differences were slightly worse, but less than 10 and 60 mm. However, maximum residuals for ITRF2000, ITRF2005 and ITRF2008 in some cases exceeded 10 and 60 mm in the horizontal and vertical components, respectively, and were as large as 17 mm in east and 84 mm in the vertical.

For ITRF1996, much larger magnitude residuals, up to 126.1 and 192.9 mm were found in the horizontal and vertical components, respectively. This was attributed to the Townsville and Karratha stations, which are poorly determined in ITRF1996 due to their limited observation span. Similarly for ITRF1997, maximum residuals of 48.8 and 464.2 mm were found in the horizontal and vertical components, respectively, with the worst fitting station being Darwin, which was also poorly determined in ITRF1997 and particularly in vertical velocity (i.e. 0.10 ± 0.64 m yr⁻¹). These large residuals had little impact on the computed transformation parameters given their large formal uncertainty (i.e. ×10 larger than what is typical), so they were not removed from the estimation process.

The major results of this paper, the final parameters and their uncertainties, are given in Table 2. To support user implementation, a fully worked example is provided in Appendix A and the approach as applied to differential GNSS vectors is shown in Appendix B.

<table>
<thead>
<tr>
<th>Frame</th>
<th>Root-mean-square ( RMS) difference (mm)</th>
<th>Maximum Residual (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITRF 2008</td>
<td>5.1</td>
<td>27.8</td>
</tr>
<tr>
<td>ITRF 2005</td>
<td>4.4</td>
<td>29.5</td>
</tr>
<tr>
<td>ITRF 2000</td>
<td>3.3</td>
<td>54.8</td>
</tr>
<tr>
<td>ITRF 1997</td>
<td>25.8</td>
<td>178.8</td>
</tr>
<tr>
<td>ITRF 1996</td>
<td>22.1</td>
<td>90.1</td>
</tr>
<tr>
<td>ITRF 1997</td>
<td>9.7</td>
<td>51.4</td>
</tr>
<tr>
<td>ITRF 2005</td>
<td>10.3</td>
<td>61.0</td>
</tr>
<tr>
<td>ITRF 2000</td>
<td>4.9</td>
<td>83.6</td>
</tr>
<tr>
<td>ITRF 1997</td>
<td>48.8</td>
<td>464.2</td>
</tr>
<tr>
<td>ITRF 1996</td>
<td>48.6</td>
<td>192.9</td>
</tr>
</tbody>
</table>

5.1. Assessment

To assess the quality of the transformation approach 126 independent weekly network solutions (January 2008 to June 2010), publicly available from Geoscience Australia (GA), were transformed from ITRF2005 to GDA94 and subsequently compared them to the official gazetted GDA94 coordinates at the AFN stations. These GA solutions, which are the Operational Final Products produced by the National Geospatial Reference Systems (NGRS) Project, were derived using the Bernese GPS Software Version 5.0 (Hugentobler et al. 2005) and incorporate data from the Australian Regional GPS Network (ARGN) (Twilley and Digney 2001), the IGS network in the Asia-Pacific region and the AuScope network (AuScope Geospatial Team 2008). Satellite orbits and EOPs were fixed to the IGS final products, which for the solutions tested were expressed in the IGS05 realisation of ITRF2005. Station coordinates were estimated once every 24 hours and the datum was defined using a free-network approach.
and tropospheric delays estimated every hour and horizontal gradients in the N-S and E-W directions were estimated once every 24 hours. The site displacement modelling was consistent with the IERS Conventions 2003 (McCarthy and Petit 2004) and ocean tidal loading corrections were applied according to the GOT00.2 model, which is an updated version of the GOT99 model (Ray 1999). Satellite and antenna phase centre corrections were applied from the IGS05 model (Schmid et al. 2007). Carrier phase ambiguities were resolved in a baseline by baseline mode using the Quasi-Ionosphere-Free (QIF) approach (Mervart and Schäer 1994). The daily solutions were combined into a weekly solution at the normal equation level and subsequently converted into SINEX format.

Overall, the RMS of the transformed weekly solutions, with respect to the combined solution, was less than 2 mm and 5 mm in the horizontal and vertical components, respectively, Table 4. The RMS difference of the transformed coordinates with respect to the gazetted coordinates was 8.4, 17.5, 36.4 mm in the north, east and up components, respectively. Of concern, however, were the large residuals in the vertical component at the Perth and Yarragadee stations. For the Perth station the residual is well explained by subsidence observed at this site (i.e. \(-6 \text{ mm yr}^{-1}\) in ITRF2005, Table 3), which has previously been suggested to be associated with groundwater extraction (Jia et al. 2007). However, the Yarragadee residual is not understood at this time, but may indicate error in its GDA94 coordinate. These large vertical residuals motivate the development of localised deformation models, which would be used to improve GDA94 recovery in the vicinity of these stations, however, this is beyond the scope of this work.

Table 2: ITRF-to-GDA94 transformation parameters and their uncertainties (1\(\sigma\)). Units are mm and mm yr\(^{-1}\) for the translations and their rates, respectively, parts-per-billion (ppb) and ppb yr\(^{-1}\) for scale and its rate, respectively, and milli-arc-seconds (mas) and mas yr\(^{-1}\) for the rotations and their rates, respectively. The parameter estimates, after unit conversion, can be substituted into equation (1) as appropriate. The reference epoch, \(t_0\), was 1994.

<table>
<thead>
<tr>
<th>From ITRF2008 to GDA94</th>
<th>(t_x), (t_y)</th>
<th>(t_z), (s_x)</th>
<th>(s_y)</th>
<th>(r_x), (r_y)</th>
<th>(r_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pm)</td>
<td>-84.68</td>
<td>32.01</td>
<td>9.710</td>
<td>-0.4254</td>
<td>2.2578</td>
</tr>
<tr>
<td>Rates</td>
<td>0.91</td>
<td>1.06</td>
<td>0.126</td>
<td>0.0221</td>
<td>0.0236</td>
</tr>
<tr>
<td>(\pm)</td>
<td>0.08</td>
<td>0.90</td>
<td>0.109</td>
<td>1.5461</td>
<td>1.1820</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>From ITRF2005 to GDA94</th>
<th>(t_x), (t_y)</th>
<th>(t_z), (s_x)</th>
<th>(s_y)</th>
<th>(r_x), (r_y)</th>
<th>(r_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pm)</td>
<td>-79.73</td>
<td>38.03</td>
<td>6.636</td>
<td>-0.0351</td>
<td>2.1211</td>
</tr>
<tr>
<td>Rates</td>
<td>2.56</td>
<td>3.37</td>
<td>0.227</td>
<td>0.0883</td>
<td>0.0972</td>
</tr>
<tr>
<td>(\pm)</td>
<td>2.25</td>
<td>-0.56</td>
<td>0.294</td>
<td>1.4707</td>
<td>1.1443</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>From ITRF2000 to GDA94</th>
<th>(t_x), (t_y)</th>
<th>(t_z), (s_x)</th>
<th>(s_y)</th>
<th>(r_x), (r_y)</th>
<th>(r_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pm)</td>
<td>-45.91</td>
<td>20.37</td>
<td>7.070</td>
<td>-1.6705</td>
<td>0.4594</td>
</tr>
<tr>
<td>Rates</td>
<td>8.74</td>
<td>11.05</td>
<td>0.423</td>
<td>0.2852</td>
<td>0.3602</td>
</tr>
<tr>
<td>(\pm)</td>
<td>-4.66</td>
<td>11.24</td>
<td>0.249</td>
<td>1.7454</td>
<td>1.4868</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>From ITRF1997 to GDA94</th>
<th>(t_x), (t_y)</th>
<th>(t_z), (s_x)</th>
<th>(s_y)</th>
<th>(r_x), (r_y)</th>
<th>(r_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pm)</td>
<td>-14.63</td>
<td>25.32</td>
<td>6.695</td>
<td>-1.7893</td>
<td>-0.6047</td>
</tr>
<tr>
<td>Rates</td>
<td>11.07</td>
<td>14.27</td>
<td>0.425</td>
<td>0.3757</td>
<td>0.4642</td>
</tr>
<tr>
<td>(\pm)</td>
<td>-8.60</td>
<td>11.25</td>
<td>0.007</td>
<td>1.6394</td>
<td>1.5198</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>From ITRF1996 to GDA94</th>
<th>(t_x), (t_y)</th>
<th>(t_z), (s_x)</th>
<th>(s_y)</th>
<th>(r_x), (r_y)</th>
<th>(r_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pm)</td>
<td>24.54</td>
<td>-68.12</td>
<td>6.901</td>
<td>-2.7359</td>
<td>-2.0431</td>
</tr>
<tr>
<td>Rates</td>
<td>20.33</td>
<td>28.66</td>
<td>0.653</td>
<td>0.7627</td>
<td>0.9049</td>
</tr>
<tr>
<td>(\pm)</td>
<td>-21.80</td>
<td>26.27</td>
<td>0.388</td>
<td>2.0203</td>
<td>2.1735</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(t_x), (t_y)</th>
<th>(t_z), (s_x)</th>
<th>(s_y)</th>
<th>(r_x), (r_y)</th>
<th>(r_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pm)</td>
<td>7.53</td>
<td>10.74</td>
<td>0.228</td>
<td>0.2850</td>
</tr>
</tbody>
</table>
Table 3: Estimated vertical velocity and integrated displacement (1994 to 2010, corresponding to a 16 year time span) and their associated uncertainty (1σ) at the AFN stations (also including Perth), derived from ITRF2005 (Altamimi et al. 2007). Significant subsidence (approximately 0.1 m) was observed at the Perth station.

<table>
<thead>
<tr>
<th>Station</th>
<th>Vertical velocity (mm yr⁻¹)</th>
<th>Deformation 1994 to 2010 (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice Springs</td>
<td>−0.4 ± 0.3</td>
<td>−6 ± 4.8</td>
</tr>
<tr>
<td>Ceduna</td>
<td>−2.0 ± 0.3</td>
<td>−32 ± 4.8</td>
</tr>
<tr>
<td>Darwin</td>
<td>−1.0 ± 0.4</td>
<td>−16 ± 6.4</td>
</tr>
<tr>
<td>Hobart</td>
<td>0.6 ± 0.2</td>
<td>10 ± 3.2</td>
</tr>
<tr>
<td>Karratha</td>
<td>0.8 ± 0.3</td>
<td>13 ± 4.8</td>
</tr>
<tr>
<td>Perth</td>
<td>−6.0 ± 0.3</td>
<td>−96 ± 4.8</td>
</tr>
<tr>
<td>Tidbinbilla</td>
<td>−0.2 ± 0.2</td>
<td>−3 ± 3.2</td>
</tr>
<tr>
<td>Townsville</td>
<td>1.2 ± 0.3</td>
<td>19 ± 4.8</td>
</tr>
<tr>
<td>Yarragadee</td>
<td>1.0 ± 0.1</td>
<td>16 ± 4.8</td>
</tr>
</tbody>
</table>

Table 4: The RMS of the weekly station coordinates (126 × 7-day solutions), with respect to the combined solution, after transformation to GDA94 and the mean residual with respect to their gazetted positions.

<table>
<thead>
<tr>
<th>Station</th>
<th>ITRF-to-GDA94 transformed coordinate RMS</th>
<th>Mean residual transformed – gazetted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>North (mm) East (mm) Up (mm)</td>
<td>North (mm) East (mm) Up (mm)</td>
</tr>
<tr>
<td>Alice Springs</td>
<td>1.5 1.0 4.3</td>
<td>0.0 8.9 −13.1</td>
</tr>
<tr>
<td>Ceduna</td>
<td>1.5 1.3 3.3</td>
<td>−0.5 10.8 18.2</td>
</tr>
<tr>
<td>Darwin</td>
<td>1.8 1.6 6.6</td>
<td>1.5 −14.7 4.8</td>
</tr>
<tr>
<td>Hobart</td>
<td>1.9 1.4 3.5</td>
<td>−16.5 8.1 11.4</td>
</tr>
<tr>
<td>Karratha</td>
<td>1.9 1.4 3.6</td>
<td>8.4 8.1 −22.9</td>
</tr>
<tr>
<td>Perth</td>
<td>2.5 3.5 6.9</td>
<td>11.9 26.8 −68.4</td>
</tr>
<tr>
<td>Tidbinbilla</td>
<td>1.7 1.3 3.3</td>
<td>4.0 5.6 −6.7</td>
</tr>
<tr>
<td>Townsville</td>
<td>1.6 1.6 3.9</td>
<td>−8.0 −3.9 −10.2</td>
</tr>
<tr>
<td>Yarragadee</td>
<td>1.7 1.5 5.4</td>
<td>−0.9 −33.9 67.9</td>
</tr>
<tr>
<td>All sites</td>
<td>1.8 1.8 4.7</td>
<td>8.4 17.5 36.4</td>
</tr>
</tbody>
</table>

6. Discussion

In this section, to further demonstrate the need and applications of these transformations, the effect of plate motion on GDA94 differential vectors is reviewed to highlight the importance of using consistent satellite trajectories and reference station coordinates. Also, as an alternative to station coordinate transformation, satellite trajectory transformations are investigated to quantify the limitations of this approach. Finally, a number of arguments towards the modernisation of the geocentric datum of Australia are illustrated.

6.1. Plate motion and GDA94

Soler (2001) highlights the importance when undertaking differential, or baseline, GNSS analysis of maintaining a consistency between reference station coordinates and the satellite trajectories; However, anecdotal evidence suggests that many Australian users adopt precise IGS or WGS84 satellite trajectories concurrently with reference station coordinates expressed with respect to GDA94. A specific concern of this practice is the time-dependent variations in the orientation of differential vectors, which result because the relative motion of stations on the Australian Plate over time are poorly modelled as a simple 3 component translations. The effect is illustrated on a geological time-scale in Figure 2 and is also significant for geodetic applications particularly as both the inter-station distance and the time...
elapsed relative to the GDA94 reference epoch (i.e. 1994) increases. These baseline differences are typically expressed in both the horizontal and vertical coordinate components and depend on the baseline’s orientation as well as the station’s position on the Australian Plate.

To demonstrate the importance of modelling this effect with an appropriate transformation methodology, a number of arbitrary inter-station baselines were calculated at the reference epoch of GDA94 (i.e. 1994). Then using a rigid plate velocity model (Altamimi et al. 2007) the station coordinates were projected to the year 2010 and re-calculated the baselines. While dependent on their orientation, the differences in the differential baselines could exceed 10 mm in magnitude for baselines as short as 70 km and were, in some cases, greater than 100 mm for baselines of 1000 km length, Table 5.

### Table 5: Maximum differences for coordinates derived from GDA94 baselines versus those derived from present ITRF baselines (i.e. 2010.0) for various inter-station distances. The difference may exceed 10 mm for baselines as short as 70 km.

<table>
<thead>
<tr>
<th>Baseline length (km)</th>
<th>Maximum horizontal difference (mm)</th>
<th>Maximum vertical difference (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>70</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>150</td>
<td>14</td>
<td>26</td>
</tr>
<tr>
<td>300</td>
<td>28</td>
<td>53</td>
</tr>
<tr>
<td>500</td>
<td>75</td>
<td>88</td>
</tr>
<tr>
<td>1000</td>
<td>100</td>
<td>175</td>
</tr>
</tbody>
</table>

To demonstrate the importance of modelling this effect with an appropriate transformation methodology, a number of arbitrary inter-station baselines were calculated at the reference epoch of GDA94 (i.e. 1994). Then using a rigid plate velocity model (Altamimi et al. 2007) the station coordinates were projected to the year 2010 and re-calculated the baselines. While dependent on their orientation, the differences in the differential baselines could exceed 10 mm in magnitude for baselines as short as 70 km and were, in some cases, greater than 100 mm for baselines of 1000 km length, Table 5.

#### 6.2. Orbit and EOP transformations to GDA94

The transformation of satellite trajectories and Earth Orientation Parameters (EOPs) are an alternative to station coordinate transformations (e.g., Kouba 2002). This approach potentially simplifies access to GDA94 by allowing users to work exclusively in a non-ITRF local datum. To assess this approach, the results derived from an Australian network processed conventionally with ITRF based products then subsequently transformed to GDA94 was compared to an approach where the analysis was undertaken using transformed orbit and EOPs.

To generate a reference solution the procedures as described in Section 5.1 were adopted. Coordinates were estimated for a regional network of 40 stations, including all the ARGN stations from 1 week of data, i.e. days 181 to 187, 2009 (gpsweek 1486). The weekly combined coordinates were transformed to GDA94 using the transformation parameters computed in this work.

To transform the IGS final orbits and EOPs to GDA94 the trnfsp3n software (Kouba 2002) was used. After updating the apriori station coordinates to GDA94, but otherwise adopting the identical computation procedures and standards to those of the reference solution, the test GDA94 solution was computed. The RMS differences of the daily station coordinates, with respect to the combined weekly solution, was 2.1, 2.8 and 7.2 mm in the north, east and vertical components, respectively, which was only slightly worse than those of the reference solution. However, there were RMS differences of 2.2, 1.5 and 53.5 mm in the north, east and vertical components, respectively, between this weekly solution and the reference solution. The large RMS values of the vertical component, which had a mean of 53.3 mm, reflects the scale bias (8.2 ppb) between GDA94 and the GPS observations themselves.

In a further test, and in an attempt to reduce this scale bias, the IGS satellite trajectories were arbitrarily rescaled. As expected, the scale change of the trajectories did not have a direct relationship with the scale of the ground network and also negatively impacted the solution quality, for example, it reduced the carrier-phase ambiguity estimation success. To further explore the impact of rescaling, the test solution computations were repeated without applying the ITRF-to-GDA94 scale and scale-rate parameters in the transformation of the orbit products. For this solution, the RMS differences of the daily station coordinates with respect to the weekly solution was an improved 1.9, 2.3 and 7.2 mm in the north, east and vertical components, respectively. This improvement reflects a greater consistency between the orbit products and the observations. With respect to the reference solution, the RMS differences were 1.2, 0.8 and 64.4 mm in the north, east and vertical components, respectively, which is an improvement in the horizontal component repeatability, but also an increase in the scale bias of the station network with respect to GDA94.

Consequently, the ITRF-to-GDA94 trajectory and EOP transformation approach might be used for GDA94 computations, but the approach is limited by the scale difference between GDA94 and the recent ITRF realisations. These differences particularly reflect on the applicability of the technique for long baseline measurements (i.e. 500 km and greater). Overall, for 300 km baselines the differ-
ences between the two approaches was less than 3 mm in total and for 100 km baselines they were less than 1 mm. This suggests that the orbit transformation approach would be suitable for applications associated with most Australian Continuously Operating Reference Station (CORS) networks, which typically have an inter-station separation of approximately 70 km or less. Given interest from users, Geoscience Australia may in the future routinely produce GDA94 based satellite trajectory and Earth Orientation products based on the IGS products.

6.3. A modernised datum for Australia

Five major drivers motivate the need for a new Australian datum. First, are the significant residual coordinate differences between the GDA94 coordinates at the AFN (i.e. the recognised-value standard for position) and the most recent transformed state-of-the-art GPS analyses transformed to GDA94. These differences, which for ITRF2008 are as large as 12 and 51 mm in the horizontal and vertical components, respectively, mean that deriving GDA94 coordinates in a consistent and nation-wide basis and meeting user expectations for accuracy (i.e. 1 cm level positioning) is not currently possible. Second, is the current uncertainty of GDA94 at the AFN (i.e. horizontal 30 mm and vertical 50 mm at the 95% confidence level) which sets the ultimate limit of uncertainty of all geodetic infrastructure in Australia. Consequently, uncertainties rigorously propagated from the AFN down through State and Territory survey control often fail to meet many user’s expectations, a view being reinforced by state-of-the-art GPS analyses that produce station coordinate repeatability at the 1–2 and 3–5 mm level in the horizontal and vertical components, respectively. Third, is the increasing divergence of ITRF and GDA which demands that users adopt careful transformation procedures to avoid positioning errors associated with not only the absolute differences but also with small changes in the orientation of differential baselines due to the rotation of the Australian Plate since 1994. As these differences increase a greater number of less sophisticated users will be required to apply coordinate transformations which can be a source of confusion and inconsistent treatment. Fourth, is the recent development of national CORS infrastructure and in particular the development of the AuScope network that provides a new opportunity for densification of the recognised-value standard for position from the 8 AFN stations used to realise GDA94 to over 100 stations. A densification of the recognised-value standard would provide improved access to and robustness of the legal traceability of position measurement in Australia. Finally, discontinuities observed in present day, GPS, coordinate time series caused by changes in the firmware, receiver, antenna, monument, or radome (e.g., Tregoning et al. 2004, Williams 2003) together with ongoing deformation of the Australian Crust in response to geophysical (e.g., Dawson et al. 2008, Watson et al. 2010), anthropogenic (e.g., Jia et al. 2007) and hydrological processes (e.g., Tregoning et al. 2009) means that for high accuracy positioning a static geodetic model for station coordinates can no longer meet the requirements of all users.

7. Conclusions

In this work new and improved ITRF-to-GDA94 transformations for a range of realisations are provided including the latest realisation, ITRF2008. They are to be adopted by Geoscience Australia for coordinate transformations applied in its capacity as Verifying Authority of Position Measurement in accordance with the National Measurement Regulations 1999, National Measurement Act 1960, and the relevant requirements of the National Association of Testing Authorities (NATA), Australia. The large residuals observed in Western Australia and in particular for the Perth Metropolitan region, which has experienced significant subsidence since 1994, motivate the need for the future development of time-dependent deformation modelling to support geodetic positioning. Significant residual differences between the gazetted GDA94 and ITRF2008 coordinates at the AFN are a major impediment to nationally-consistent centimetre-level accuracy positioning. Additionally, left unaddressed, the link between GDA94 position measurements and the International System of units (SI units) through the ITRF will become increasingly tenuous. Improving the link between Australia’s national datum and the ITRF, to meet the demands of precision positioning applications while also having minimal broader user impact, might be achieved by updating the recognised-value standard for position, including all the ARGU and AuScope stations, to an ITRF2008 based reference frame at a reference epoch of 1994. However, in the longer-term the inherent limitations of static coordinate datums will demand consideration of a dynamic Australian datum.
Appendix A – sample transformation

To assist those who may wish to implement the ITRF-to-GDA94 transformations in software, a fully worked example calculation is provided below. The transformation to GDA94 can be completed in a 4 step process starting with an ITRF coordinate, which in this example is a coordinate in ITRF2005 with a reference epoch of 16 June 2010, i.e.:

\[
\begin{pmatrix}
X_{ITRF} \\
Y_{ITRF} \\
Z_{ITRF}
\end{pmatrix} = \begin{pmatrix}
-4052052.3678 \\
4212836.0411 \\
-2545105.1089
\end{pmatrix},
\]

which expressed as a geodetic coordinate is:

\[
\begin{pmatrix}
\lambda_{ITRF} \\
\phi_{ITRF} \\
h_{ITRF}
\end{pmatrix} = \begin{pmatrix}
133° 53' 7.86712'' \\
-23° 40' 12.41482'' \\
603.2562
\end{pmatrix},
\]

where \(\lambda_{ITRF}\) is the station’s longitude, \(\phi_{ITRF}\) is the latitude and \(h_{ITRF}\) is the ellipsoidal height (m) relative to the GRS80 ellipsoid.

Step 1 – time elapsed to the epoch of interest

Determine the day of year (doy) of the input coordinate, e.g., day 1 corresponds to 1 January and, for this computation, 16 June 2010 corresponds to day 167 (i.e. epoch of interest). Subsequently, compute the time elapsed from the reference epoch (i.e. \(t_0 = 1994\)) to the epoch of interest. The epoch of interest is computed as \(t = 2010 + (167 - 0.5)/365.25 = 2010.4559\), where the 0.5 days subtracted from the doy is a 12 hour correction to bring the time epoch to the middle of the day. Consequently, the time elapsed between the reference epoch and the epoch of interest is \(t - t_0 = 2010.4559 - 1994 = 16.4559\) years.

Step 2 – parameter projection to the epoch of interest

Project the parameters, Table 2, to the epoch of interest, as follows:

\[
\begin{align*}
T_x &= t_x + \dot{t}_x(t - t_0) \\
&= -79.73 + 2.25 \times 16.4559 = -42.70 \text{ mm} \\
T_y &= t_y + \dot{t}_y(t - t_0) = -17.06 \text{ mm} \\
T_z &= t_z + \dot{t}_z(t - t_0) = 28.81 \text{ mm} \\
S_c &= S_c + \dot{S}_c(t - t_0) = 11.474 \text{ ppb} \\
R_x &= r_x + \dot{r}_x(t - t_0) = 24.1665 \text{ mas} \\
R_y &= r_y + \dot{r}_y(t - t_0) = 20.9515 \text{ mas} \\
R_z &= r_z + \dot{r}_z(t - t_0) = 21.3961 \text{ mas}
\end{align*}
\]

Step 3 – unit conversions

Apply unit conversion factors to the projected parameters to make them consistent with equation (1). The conversion factors for the translations: 1 mm is \(1 \times 10^{-3}\) m; scale: 1 ppb is \(1 \times 10^{-9}\) parts; and rotations: 1 mas is \((1 \times 10^{-3} \times \pi)/(360 \times 180)\) rad, and leads to:

\[
\begin{align*}
T_x &= -0.04270 \text{ m} \\
T_y &= -0.01706 \text{ m} \\
T_z &= 0.02881 \text{ m} \\
S_c &= 1.1474 \times 10^{-8} \text{ parts} \\
R_x &= 1.17163 \times 10^{-7} \text{ rad} \\
R_y &= 1.01576 \times 10^{-7} \text{ rad} \\
R_z &= 1.03731 \times 10^{-7} \text{ rad}.
\end{align*}
\]

Step 4 – transformation

Substitute the projected, unit-converted parameters into equation (1) and evaluate e.g.,

\[
\begin{pmatrix}
X_{GDA94} \\
Y_{GDA94} \\
Z_{GDA94}
\end{pmatrix} = \begin{pmatrix}
T_x \\
T_y \\
T_z
\end{pmatrix} + (1 + S_c) \begin{pmatrix}
1 & R_z & -R_y \\
-R_z & 1 & R_x \\
R_y & -R_x & 1
\end{pmatrix} \begin{pmatrix}
X_{ITRF} \\
Y_{ITRF} \\
Z_{ITRF}
\end{pmatrix},
\]

which gives the GDA94 Cartesian coordinate, i.e.

\[
\begin{pmatrix}
X_{GDA94} \\
Y_{GDA94} \\
Z_{GDA94}
\end{pmatrix} = \begin{pmatrix}
-4052051.7615 \\
4212836.1945 \\
-2545106.0145
\end{pmatrix},
\]

and is equivalent to the GDA94 geodetic coordinate (GRS80 ellipsoid) i.e.

\[
\begin{pmatrix}
\lambda_{GDA94} \\
\phi_{GDA94} \\
h_{GDA94}
\end{pmatrix} = \begin{pmatrix}
133° 53' 7.84795'' \\
-23° 40' 12.44581'' \\
603.3361
\end{pmatrix},
\]

where \(\lambda_{GDA94}\) longitude, \(\phi_{GDA94}\) is latitude and \(h_{GDA94}\) is ellipsoidal height (m) relative in GDA94.

Appendix B – differential vector transformations

The transformation approach can also be applied to differential GNSS vectors provided the reference frame associated with the satellite trajectory used in the analysis, its epoch, and the approximate geocen-
tric coordinate of the stations involved are known. For example, a differential vector in GDA94, $[\Delta X_{GDA94}, \Delta Y_{GDA94}, \Delta Z_{GDA94}]^T$, can be calculated from an ITRF baseline, $[\Delta X_{ITRF}, \Delta Y_{ITRF}, \Delta Z_{ITRF}]^T$, using equation (2) where $T\{ \}$ is the ITRF-to-GDA94 transformation and $[X_0, Y_0, Z_0]^T$ is the approximate geocentric coordinate of the starting station.

$$
\begin{bmatrix}
\Delta X_{GDA94} \\
\Delta Y_{GDA94} \\
\Delta Z_{GDA94}
\end{bmatrix}
= T
\begin{bmatrix}
X_0 + \Delta X_{ITRF} \\
Y_0 + \Delta Y_{ITRF} \\
Z_0 + \Delta Z_{ITRF}
\end{bmatrix}
- T
\begin{bmatrix}
X_0 \\
Y_0 \\
Z_0
\end{bmatrix}
$$

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